Free-falling dust ball universe: Expanding space and intensifying gravity

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Big bang, inflation, RW metric and Λ-CDM model is widely accepted current standard cosmological model. However, many questions are left unexplained. Enormous studies have been done to settle them by modifying Einstein field equations (EFE). Modifications include energy-momentum pseudotensor, dark matter, dark energy and modified gravity such as $f_{(R)}$ gravity. But they seem to be far from convincing. Here we show that those questions can be explained without tweaking original EFE at all by a new interpretation of general relativity (GR). We found that any objects placed where the time is dilated shrink smaller as the wave length of light becomes shorter. Therefore, if we are in such a place, we shrink and space is observed spacious. Thus, we defined a new metric named *metric for matter* for shrunken observer, and made cosmological study with it. We carried out some thought experiments to show the validity of the new metric. Furthermore, we developed an original simulation code which can work under extremely strong relativistic situation. With it, we showed that our new hypothesis of *Free-falling dust ball universe* can answer those questions.

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Introduction

There are two ways to tune Einstein field equations (EFE) to fit it to observation result. One way is to modify right hand side of the equations which represents the matter and energy content. Energy-momentum pseudotensor was introduced to settle energy conservation problem of GR. Cold dark matter (CDM) was introduced to explain gravitational effects observed in very large structures such as anomalies in the rotation of galaxies, enhanced clustering of galaxies that cannot be explained by the quantity of observed matter. Cosmological constant Λ, also known as dark energy, was reintroduced to explain current accelerating expansion of the universe.

The other way is to tune left hand side of the equations which represents the curvature of spacetime which produces gravity. Modifications to gravity such as $f_{(R)}$ gravity, scalar-tensor theories, Gauss-Bonnet gravity, the Dvali-Gabadazde-Porrati braneworld and Galileon gravity have received much attention recently [1]. However, all of these can be criticised as artificial parameterisations.

Here we propose third way to explain observation result. We do not tweak EFE at all. Instead, we introduce a new interpretation of EFE which employs *metric for matter*, and with it, we propose a new hypothesis about evolution of the universe.

METRIC FOR MATTER

Suppose there is a spherical thin shell whose radius is R, mass is M, Schwarzschild radius is a as shown in Fig. 1B and 1D. The geometry just outside the shell M (place 2) is Schwarzschild geometry whose line element ds at radius r is given by:

$$ds^2 = -\frac{1}{\gamma^2}(cdt)^2 + \gamma^2 dr^2 + (rd\Omega)^2 \tag{1}$$

$$\gamma = \left(1 - \frac{a}{r}\right)^{-1/2} \tag{2}$$

where, $rd\Omega$ denotes circumferential distance. Using γ defined by eq. 2, proper time $d\tau$ defined with the Lagrangian coordinates, i.e. coordinates which are fixed to the matter and move with it, is given by:

$$d\tau = \frac{1}{\gamma}dt \tag{3}$$

According to the Birkhoff's theorem, gravitational acceleration exerted by the mass of spherical shell vanishes inside it and spacetime becomes Minkowski's. Considering continuity of the metric toward circumferential direction, the ratio of proper distance, which is defined with Lagrangian coordinates, and Eulerian distance, which is defined with coordinates for an observer at infinity, inside the shell must be 1 regardless of the direction. Thus, we can employ Cartesian coordinates of t, x, y, z for Eulerian coordinates and τ, X, Y, Z for Lagrangian coordinates at place 3 shown in Fig. 1B. Conventional line element ds inside the shell is given by:

$$\begin{aligned} ds^2 &= -(cd\tau)^2 + dX^2 + dY^2 + dZ^2 \\ &= -\frac{1}{\gamma^2}(cdt)^2 + dx^2 + dy^2 + dz^2 \end{aligned} \tag{4}$$

Usually, papers concerning GR are written in abstract mathematical manner. However, putting aside mathematical elegance, we give priority to intuitive understanding. For this reason, we look into some concrete examples. For instance, we work with $\gamma = 2$ world. With eq. 2, γ value at the surface of the shell becomes 2 when $R = (4/3)a$, and considering the continuity, γ value of eq. 4 becomes 2 everywhere inside the shell regardless of the direction. There, we calculate the diameter of a hydrogen atom D with Bohr model. Symbols with subscript "1" and "3" indicate place 1 and 3 in Fig. 1B, "E" and "L" indicate physical value defined with Eulerian and Lagrangian coordinates respectively.

$$c_{3L} = dX/d\tau = c_{0L} = c_0 \tag{5}$$

$$c_{3E} = dx/dt = \frac{1}{\gamma}c_{0L} = \frac{1}{\gamma}c_0 \tag{6}$$

Where, c_0 denotes speed of light at infinity. By the principle of equivalence, de Broglie's wavelength λ at place 1 and 3 must be the same with Lagrangian coordinates.

$$\lambda_{3L} = \lambda_{1L} \tag{7}$$

Then, with Eulerian coordinates,

$$\lambda_{3E} = \frac{1}{\gamma}\lambda_{1E} \tag{8}$$

$$\frac{D_{3E}}{D_{1E}} = \frac{\lambda_{3E}}{\lambda_{1E}} = \frac{1}{\gamma} \tag{9}$$

Eq. 9 shows that size of a hydrogen atom D in $\gamma = 2$ world becomes a half of its original size with Eulerian coordinates. Deductively, if we put a copy of the earth at place 3 in Fig. 1B, speed of a ball thrown by a pitcher becomes a half measured with Eulerian coordinates as the speed of light becomes a half. At the same time, as the size of a hydrogen atom shrinks, distance from pitcher's mound to the catcher becomes a half. Therefore, flying time of the ball keep the same measured both with Eulerian and Lagrangian coordinates.

It is often said that time freezes near the surface of a black hole. Whereas, this thought experiment shows that time for any matter proceed at the same speed regardless of γ value. Considering this phenomena, we propose a new line element *material line element* in place of eq. 4 as follows.

$$ds^2 = -(cdt)^2 + \gamma^2(dx^2 + dy^2 + dz^2) \tag{10}$$

For Schwarzschild geometry, material line element is presented as follows in place of eq. 1.

$$ds^2 = -(cdt)^2 + \gamma^4 dr^2 + \gamma^2(rd\Omega)^2 \tag{11}$$

We introduce a new metric tensor the *metric for matter* defined as coefficients of eq. 10, 11.

In GR, length of time and space are expressed with a set of values and a metric tensor. If we change the metric, corresponding values change. However, they express the same thing. Therefore, we can use whatever metric we like. For dealing with matter which shrinks as stated above, the metric for matter is convenient. For dealing with light, conventional metric is convenient. Therefore, we call it the *metric for light* in this study to distinguish them explicitly.

THOUGHT EXPERIMENTS

Thought experiment 1: Light and hydrogen atom in γ=2 world

We carry out thought experiments and express the results with Eulerian and Lagrangian coordinates. For Eulerian coordinates, Minkowski's metric is used. And for Lagrangian coordinates, the metric for light and the metric for matter are used to express the same physical meaning.

Suppose we have a spherical shell whose mass is M, Schwarzschild radius is a, and radius R is $(4/3)a$ as illustrated in Fig. 1B. γ value at the surface of the shell M (place 2) and inside the shell M (place 3) is 2 by eq. 2. From infinitely far place (place 1), green light of wave length $\lambda_G = 500$ nm is emitted for $\Delta\tau = 1$ second and irradiate place 2 radially and circumferentially.

Results are shown in Table 1. At place 2, elapsed time of the light $\Delta\tau$ measured with the metric for light becomes a half by eq. 3. Propagation distance L within the elapsed time becomes shorter measured with Eulerian coordinates. L measured with the metric for light differs from Eulerian distance according to the metric shown as coefficients of eq. 1. L measured with the metric for matter differs from Eulerian distance according to the metric shown as coefficients of eq. 11. Speed of light c on each column is calculated from $\Delta\tau$ and L on the corresponding column.

Because N_G in $\Delta\tau$ is not changed and packed into the distance L, wave length λ_G measured with Eulerian coordinates becomes shorter at place 2. This phenomenon is called gravitational blue shift. Diameter of a hydrogen atom D becomes smaller in accordance with λ_G, too. Whereas, measured with the metric for matter, all values keep the same as those of place 1.

	Place 1	Place 2					
		Radial direction			Circumferential direction		
	Eulerian	Eulerian	Metric (light)	Metric (matter)	Eulerian	Metric (light)	Metric (matter)
$\Delta\tau$ (s)	1	1	0.5	1	1	0.5	1
L $(\times 10^8 m)$	3	0.75	1.5	3	1.5	1.5	3
c $(\times 10^8 m/s)$	3	0.75	3	3	1.5	3	3
N_G $(\times 10^8)$	6	6	6	6	6	6	6
λ_G (nm)	500	125	250	500	250	250	500
D (pm)	53	13.2	26.5	53	26.5	26.5	53

TABLE I. Result of thought experiment 1. Green light of wave length $\lambda_G = 500nm$ is emitted from place 1 in Fig. 1A for 1 second and irradiate place 2 from two direction.

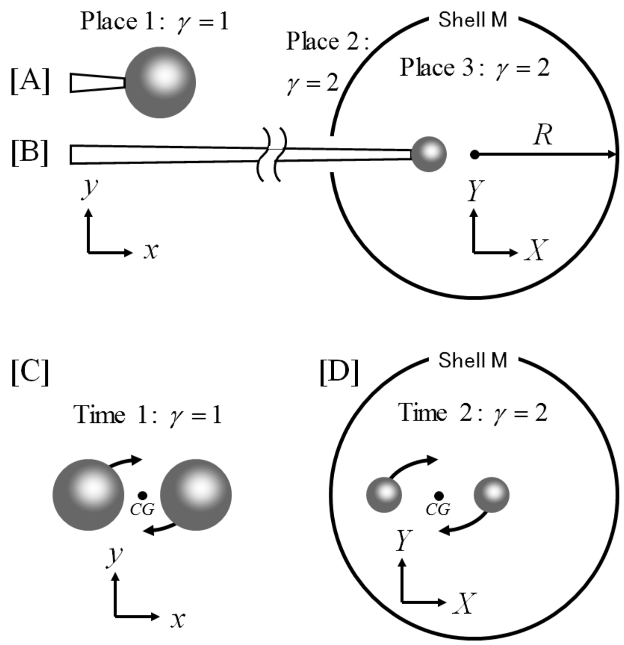

FIG. 1. Settings for thought experiments. (A) Pushing a ball of $1kg$ with a stick at infinitely far place from the Shell M (place 1). (B) Pushing the same ball in the shell M (place 3) with a very long stick from place 1. place 2 is just outside the shell M. (C) Two balls of $1kg$ are orbiting around the common CG inside a infinitely large shell M (not illustrated). (D) The same orbiting balls when the shell shrink to $R = (4/3)a$.

Thought experiment 2: Pushing balls

Suppose we push two same balls of 1 kg with sticks as shown in Fig. 1A and 1B. The size of the ball at place 3 is a half of its original viewed with Eulerian eyes. We are at the ends of the sticks at place 1 and push the ends of the sticks for 1 second. Initial velocity v of the input end is $0m/s$ and acceleration rate α is $1m/s^2$. For pushing the balls, we employ two kinds of very long rigid virtual sticks.

First one is a *light metric stick* whose length measured with Eulerian coordinates varies with the metric for light. Results are shown in Table 2A.

Length of the part of the stick at place 2 shrinks to a half. Nevertheless, the movement ΔX of the tip at place 3 keeps the same with the input movement ΔX_0 because the metric for light for length is the same at both ends by eq. 4. ΔX at place 3 measured with the metric for matter is twice as large as the value measured with Eulerian coordinates by eq.10. Elapsed time of acceleration $\Delta\tau$ at place 3 expressed with each metric is the same as that shown on the first line of the Table 1. Each physical value like velocity and acceleration are calculated from elapsed time and displacement on the same column. Gravitational mass m is heavier with Eulerian coordinates because it is a function of u^t.

Comparing the leftmost line and rightmost line in Table 2A, values do not match. This table shows that the principle of equivalence is not satisfied if we employ light metric stick.

The other kind of stick is a *material stick* whose length measured with Eulerian coordinates varies with the metric for matter. In this case, the movement of the tip is a half of the input movement because the size of the tip part of the stick shrink to a half. Physical values at each place are shown in Table 2B.

The bottom line of Table 2B shows that the energy at both ends matches right. And comparing the leftmost column and rightmost column, values match perfectly. This shows that the principle of equivalence is satisfied if we employ material stick. This result shows that our new interpretation of GR, size of matter shrink where γ value is large, is valid.

Thought experiment 3: Falling balls

Suppose we have two balls of 1 kg at place 3 in Fig. 1B. Initial distance d between them is 2 m and initial velocity of each ball is 0 m/s. They begin falling to each other by their own gravity and fall for 1 second. Results are shown in Table 3. At place 3, proportion of $\Delta\tau$ for each metric is the same as the first line of Table 2A and 2B. At place 1, final displacement ΔX_0 is calculated by Newtonian equation of motion. Symbols with subscript "0" indicate value at infinity and G denotes gravitational

[A]			Place 1	Place 3		
			Eulerian	Eulerian	Metric (light)	Metric (matter)
Elapsed time	$\Delta\tau$	(s)	1	1	1/2	1
Displacement	ΔX	(m)	1/2	1/2	1/2	1
Final velocity	$v_f = dX/d\tau$	(m/s)	1	1	2	2
Acceleration	$\alpha = v_f/\Delta\tau$	(m/s^2)	1	1	4	2
Four-velocity	$u^t = dt/d\tau$	(m/s)	c	2c	c	c
Inertial mass	$m = m_0(u^t/c)^2$	(kg)	1	4	1	1
Force	$F = m\alpha$	(N)	1	4	4	2
Energy	$E = F\Delta X$	(J)	1/2	2	2	2

[B]			Place 1	Place 3		
			Eulerian	Eulerian	Metric (light)	Metric (matter)
Elapsed time	$\Delta\tau$	(s)	1	1	1/2	1
Displacement	ΔX	(m)	1/2	1/4	1/4	1/2
Final velocity	$v_f = dX/d\tau$	(m/s)	1	1/2	1	1
Acceleration	$\alpha = v_f/\Delta\tau$	(m/s^2)	1	1/2	2	1
Four-velocity	$u^t = dt/d\tau$	(m/s)	c	2c	c	c
Inertial mass	$m = m_0(u^t/c)^2$	(kg)	1	4	1	1
Force	$F = m\alpha$	(N)	1	2	2	1
Energy	$E = F\Delta X$	(J)	1/2	1/2	1/2	1/2

TABLE II. Result of thought experiment 2. (A) A ball is pushed with a *light metric stick.* (B) A ball is pushed with a *material stick.*

constant.

Because the speed of Eulerian time progress slows to a half at place 3, ΔX becomes a quarter of original measured with Eulerian coordinates. Measured with the metric for matter, this value doubles because the scale shrinks to a half. Gravitational acceleration α_G is derived from ΔX and $\Delta\tau$ on the corresponding column. Gravitational constant G for each column is derived from m, d, α_G on the corresponding column.

Although G constant is called *fundamental physical constant (FPC)* and believed to keep "constant", and in GR, geometrized unit system where $c = 1, G = 1$ is commonly used, it depends on metric as shown in Table 3. Due to this, if the metric which rules our world changes, G constant also changes.

According to our hypothesis described later, γ value around our world is increasing. If this hypothesis is right, G constant of our world must be increasing.

Thought experiment 4: Orbiting balls

Suppose we have two balls of 1 kg orbiting around the common center of gravity (CG) inside a infinitely large shell M (not illustrated) as shown in Fig. 1C. Orbiting radius is 1 m at time 1. Because R is large enough, γ value can be assumed to be 1. At time 2, the radius of the shell shrinks to $R = (4/3)a$ and γ value inside the shell becomes 2 as shown in Fig. 1D.

Results are shown in Table IV. At time 2, position is conserved because of the law of inertia. Kinetic energy T is conserved because of the energy conservation law. From $T, m, \Delta\tau$ on the same column, orbital velocity v and centrifugal acceleration α_c for each column is obtained. If centrifugal acceleration was balanced with gravitational acceleration at time 1, α_G/α_c value on all the columns are calculated to be 1. Because right three columns express the same meaning with different metric, value of dimensionless number like α_G/α_c must be the same. Results in Table 4 satisfy this requirement. And in GR, energy is called a *scalar physical quantity* which must not vary with coordinate transformation. In Table IV, values of calculated potential energy V on all the columns keep invariant as scalar values.

According to Table IV, orbiting objects keep their orbiting radius measured with the metric for light. For an observer who lives on a copy of the earth in the huge shell M of Fig. 1D, the moon at time 2 is observed twice as high as time 1.

With this thought experiment of orbiting balls itself, dark matter can not be explained because orbits of stars does not converge to form a galaxy. Nevertheless, there are reasons by which we consider dark matter can be explained with this model.

Historically, dark matter was introduced to explain galaxy rotation curve and concentration. If stars in a galaxy are rotating steadily, rotating velocity near the center must be faster than outside. Whereas, the speed was found almost the same regardless of the radial position. And observed total mass is much lighter than the mass needed for keeping stars together.

With Newtonian dynamics, if stars are pulled together

			Place 1	Place 3		
			Eulerian	Eulerian	Metric (light)	Metric (matter)
Elapsed time	$\Delta\tau$	(s)	1	1	1/2	1
Distance	d	(m)	2	2	2	4
Displacement	ΔX	(m)	$\Delta X_0 = G_0/8$	$\Delta X_0/4$	$\Delta X_0/4$	$\Delta X_0/2$
Acceleration	α_G	(m)	$\alpha_{G0} = G_0/4$	$\alpha_{G0}/4$	α_{G0}	$\alpha_{G0}/2$
Mass	m	(kg)	1	4	1	1
G constant	G	(m^3/kgs^2)	$G_0 = 6.67 \times 10^{-11}$	$G_0/16$	G_0	$2G_0$

TABLE III. Result of thought experiment 3. Two balls are falling to each other in the shell M shown in Fig. 1B.

by their own gravity, stars gain kinetic energy. After passing the periapsis, they fly away to their original distance with its kinetic energy. Some kind of damping is required to kill the kinetic energy and keep them together. In the scale of solar system, physical contact between matter i.e. friction can do the work. Whereas, in the scale of galaxies, that is not likely because matter is so rare to have frequent physical contact.

We considered that they are not rotating steadily but on their first way heading toward the gravity source. Suppose a star is floating still at very far place from a gravity source inside a huge scale shell M. The star gains velocity by gravitational pull. By the time the star goes half way, suppose that the shell shrink and γ value doubles. The distance is evaluated as far as initial distance even though the system of the star and the gravity source has isolated from the others in the universe. If this effect represents the process of the growing perturbations of density distribution of the universe, falling matter can continue falling very long time without reaching the periapsis. This effect can explain the concentration of galaxies and large scale structures without help of unknown dark matter.

Moreover, stars initially farer from the gravity source have larger potential energy. This can explain the uniformity of the velocity.

By the way, when two or more elliptical galaxies are merging, they must be elongated by tidal force. Arms of spiral galaxies look like the tail part of elongated elliptical galaxies. If so, they are not rotating steadily but on the first way heading toward the center of the galaxies. Although this hypothesis needs further study by N-body simulation or by other ways, this is likely to be able to explain galaxy rotation anomaly and concentration.

LUNAR LASER RANGING

Lunar laser ranging revealed that the moon is receding from the earth at the rate of $3.82 \pm 0.07 cm/year$ [2]. At this rate, calculating from known parameters, the moon is gaining orbital angular momentum $\Delta P_M = 1.42 \times 10^{24} kgm^2/s$ every year. Cause of the recession is attributed to tidal force. However, if so, angular momentum of the earth-moon system must be conserved.

From recorded historical eclipses, earth's rotation speed was found to be slowing at the long term trend of $1.7ms/century$ [3]. With this rate, the earth loses its angular momentum $\Delta P_E = 1.15 \times 10^{24} kgm^2/s$ every year. This ΔP_E can raise moon only $3.06cm$ a year. We attribute the rest to the expansion of the universe caused by the change of γ value. In this case, $\dot{\gamma}/\gamma = 2.0 \times 10^{-11} year^{-1}$.

Because Table III shows that G constant is proportional to γ, change of G constant must be the same. Melnikov states that $\dot{G}/G$ compatible with modern observations does not exceeded $10^{-12} year^{-1}$ [4], which contradicts our hypothesis. We need further careful study about observation results.

GENESIS SCENARIO OF FREE-FALLING DUST BALL UNIVERSE

Argument on genesis scenario can be out of physics because no one can prove it. However, to justify the purpose of this study, we propose a new scenario. According to most accepted standard theory, our universe was created from vacuum by quantum mechanism. It began as a small particle and expanded. Initial smallness can explain homogeneity of current universe because it was small enough to mix up. Expansion stage followed. Where, there is a big question so called flatness problem. Results of observation tell us that spacetime of current universe is very flat. To achieve this flatness today, density of matter and energy in the universe at the Planck era i.e. at the beginning of the big bang, is required to be exactly the critical value with accuracy of one part in 10^{62} or less [5]. Although inflation theory was devised to explain it, it explains nothing about the mechanism. Alleged source of energy *false vacuum* does not mean anything more than "something unknown". Alleged dark energy of current universe is also literally "something unknown". Thus, the standard theory has not succeeded to explain current flatness and the principle of energy conservation at all.

In our hypothesis, we adopt first half of the standard theory. The universe was born as a small particle by quantum mechanism. It was small enough to be homogeneous. However, how could we know it was small when there was no scale to measure? Cosmological scale

			Time 1	Time 2		
			Eulerian	Eulerian	Metric (light)	Metric (matter)
Orbital radius	r	(m)	1	1	1	2
Kinetic energy	$T = mv^2/2$	(J)	T_0	T_0	T_0	T_0
Orbital velocity	$v = dX/d\tau$	(m/s)	v_0	$v_0/2$	v_0	v_0
Centrifugal acc.	$\alpha_c = -v^2/r$	(m/s^2)	α_{c0}	$\alpha_{c0}/4$	α_{c0}	$\alpha_{c0}/2$
Acc. ratio	α_G/α_c	(-)	1	1	1	1
Potential energy	$V = -Gm^2/(4r)$	(J)	$-G_0/4$	$-G_0/4$	$-G_0/4$	$-G_0/4$

TABLE IV. Result of thought experiment 4. Two balls are circling around the common CG.

is ultimately defined by balance of physical forces and time. If the physical rules were created at this point, they could be anything. Measuring the new-born particle with newly created scale, it could turn out to be billions or trillions or whatever light years across. This hypothesis seems to be stupendous. However, because it was only the change of scale, the expansion process was calm and quiet event in a moment. Compared with the fierceness of the inflation process, stupendousness of this hypothesis is not larger.

In standard model, structure of the universe is explained with Robertson-Walker (RW) model [6]. It states that although the universe has limited volume, it has no center nor outer edge. This model could be mathematically viable, however, physically, no one can imagine any concrete shape which satisfies such conditions.

In our hypothesis, we adopt much simpler dust ball universe model. The expanded dust ball began free-falling toward its center of gravity at the time of creation. Dust particles which represent stars and galaxies became congested at first, and with strong relativistic effect, as following numerical simulation shows, it began to be observed larger and more spacious from certain time point because we were getting smaller. The total energy of our universe has been conserved the same as the first particle's energy. Total energy of the current universe looks huge because we have shrunken to extremely small beings. There is a famous words, "Our universe is the ultimate free lunch". On the contrary, with our hypothesis, "Our universe is the ultimate stinginess."

NUMERICAL SIMULATION: FREE-FALLING DUST BALL UNIVERSE

Simulation code

We hit upon the idea of free-falling dust ball universe a couple of years ago and posted it on Science in June 2011. We named it "Micro Bang". However, it lacked theoretical argumentation and deservedly, it was rejected. Since then, we developed two generations of original codes for simulating radius-time two dimensional free-falling dust ball universe. As it requires capability to deal with strong relativistic phenomena, perturbation methods such as post-Newtonian approximations were ruled out. We developed codes for direct solution of EFE. The key idea for it was that in free-falling dust ball, proper time and proper distance are shared by all the dust particles. This idea was derived from reference of Suto [7].

Our first code employed rather orthodox method using geodesic equations. It worked well under weak relativistic situation. However, as we were afraid of, it blew up under strong relativistic situation. We tried various techniques to suppress the divergence, however, the effort was in vain. After a long struggle, we hit upon an idea for second generation code. We abandoned geodesic equations which are differential formalism, and instead, employed integral formalism for equations of motion.

$$\frac{du^r}{d\tau} = \frac{d^2r}{d\tau^2} = -G\frac{m(\sigma)}{r^2} \tag{12}$$

$$u^r_{(t+\Delta t,\sigma)} = u^r_{(t,\sigma)} + \frac{du^r}{d\tau}\Delta\tau \tag{13}$$

$$r_{(t+\Delta t,\sigma)} = r_{(t,\sigma)} + u^r_{(t,\sigma)}\Delta\tau \tag{14}$$

$$\phi_{(\Sigma)} = -\frac{Gm_{(\Sigma)}}{R} \tag{15}$$

$$\phi_{(\sigma)} = \phi_{(\sigma+\Delta\sigma)} + \frac{du^r}{d\tau}\Delta r \tag{16}$$

$$\gamma = u^t = \frac{dt}{d\tau} = \frac{1}{\sqrt{1+2\phi}} \tag{17}$$

σ, Σ denote radial position and radius of the dust ball in Lagrangian coordinates. $m(\sigma), \phi(\sigma)$ denote total mass of the dust inside radius σ, gravitational potential at radius σ. Gravitational potential $\phi(\Sigma)$ and γ value at the surface shell element are obtained by Schwarzschild geometry. Those values of inner shell elements are obtained from those of outer shell element by eq. 16, 17. Equation of motion corresponding with geodesic equation is eq. 12.

In general, from moving observer, gravitational mass of dust particles are observed heavier. Whereas, in this case, all particles stay still with Lagrangian coordinates. Therefore, observed from an observer free-falling with dust particles at radius σ, mass of the dust inside radius σ stays $m(\sigma)$ throughout the time measured with the metric for light. Thus, described with the metric for light, eq. 12 is valid.

This code turned out to be robust but still blew up from the outermost shell element of the dust ball. After another long struggle, we hit upon a thought experiment. Suppose we have a dust shell of mass M which has small finite thickness. Outermost dust particles are accelerated by gravity, whereas, innermost dust particles are floating in zero gravity. Hence, by natural providence, outermost particles overtake next outermost particles. For this reason, we changed our code to allow dust shell elements to overtake inner shell elements. By this amendment, our code became robust enough to simulate dynamics under extremely strong relativistic conditions.

For instance, we simulated free-falling homogeneous dust ball whose Schwarzschild radius a is 1 light year. Initial radius R is10 light years, initial radial velocity u^r is 0. The result is shown in Fig. 2. Horizontal curves represent simultaneous proper time lines where proper time τ is constant. Vertical curves represent world lines of particles where proper position σ is constant. In GR, in general, proper time and proper distance are defined locally at that particular point. However, it is known that if everything is free-falling and stays still in Lagrangian coordinates, proper time and proper space are shared by all [7].

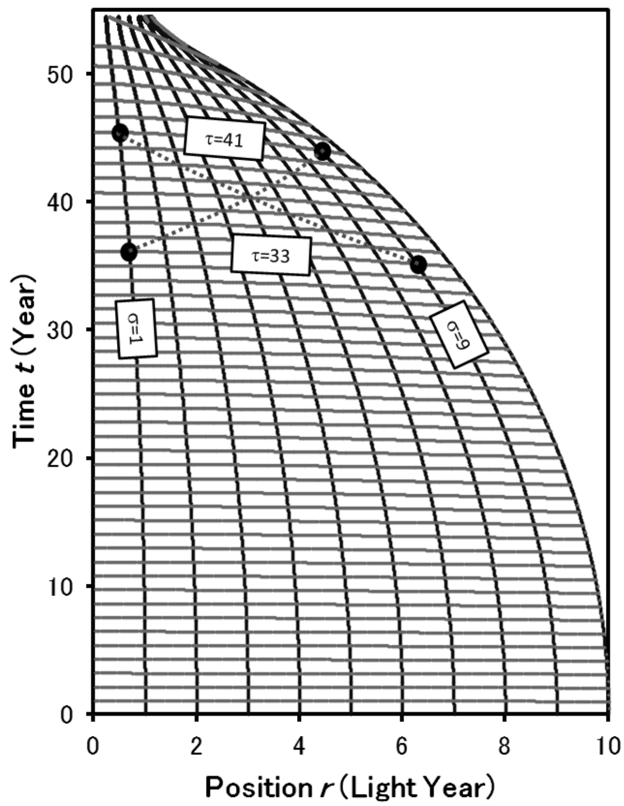

FIG. 2. Simulation result: Homogeneous dust ball of Schwarzschild radius 1 light year, initial radius 10 light years began free-falling at $t = \tau = 0$.

Simulation result

In Fig. 2, at time $t = \tau = 0$, σ is identical with r because everything is not moving. After release, distance Δr between dust particles at $\sigma = 1$ and $\sigma = 9$ contracts in Eulerian coordinates. However, with Lagrangian coordinates, $\Delta\sigma$ keeps the same value of 8 light years throughout the time. Suppose short light pulse is emitted from dust particles at $\sigma = 1$ and $\sigma = 9$ at their shared proper time $\tau = 33$ year toward the other particle. The paths of the light pulses are represented by dotted curves. They propagate from grid point to grid point because they propagate one local Lagrangian light year $\Delta\sigma$ in one local Lagrangian year $\Delta\tau$. Therefore, they reach to the other dust particle after $\Delta\tau = 8$ years for both particles simultaneously. This is what we mean by shared proper time and proper space. And for an observer who is free-falling with dust particles, the space is completely flat. This can explain the flatness of our current universe.

Every dust particle describes a parabolic line at first. However, when the sphere's radius R approaches Schwarzschild radius, falling velocity in Eulerian coordinates slows down because time progress slows. And because this effect is stronger at inner region, dust particles of outer region catch up with inner dust particles. Thus, mass are condensed to form a high density shell at outermost region. The shell continue to converge. This is similar to the situation described in Fig. 1D.

Fig. 3 is a close-up of the last part of Fig. 2. Suppose green light of wave length $\lambda_G = 500$ nm was emitted from dust particles at $\sigma = 1$ light year and $\sigma = 6$ light year from shared proper time of $\tau = 44$ to 45 year toward the other dust particle. Length of light arrows become 1 light year from both particles when $\tau = 45$ year. Total wave number $N_G = c\Delta\tau/\lambda_G$ is 1.9×10^{22} each. As the front end and rear end of the light arrows propagate one local Lagrangian light year in one local Lagrangian year, they pass through grid point to grid point.

For a stationary observer at time $t = 53.4$ year, simultaneous time line is parallel to the horizontal axis. Length of light arrow emitted from outer particle is observed shorter than one light year. However, within that length, all wave number N_G is packed in. This is observed as gravitational blue shift. And length of the light arrow from the inner particle is observed longer. This is gravitational red shift.

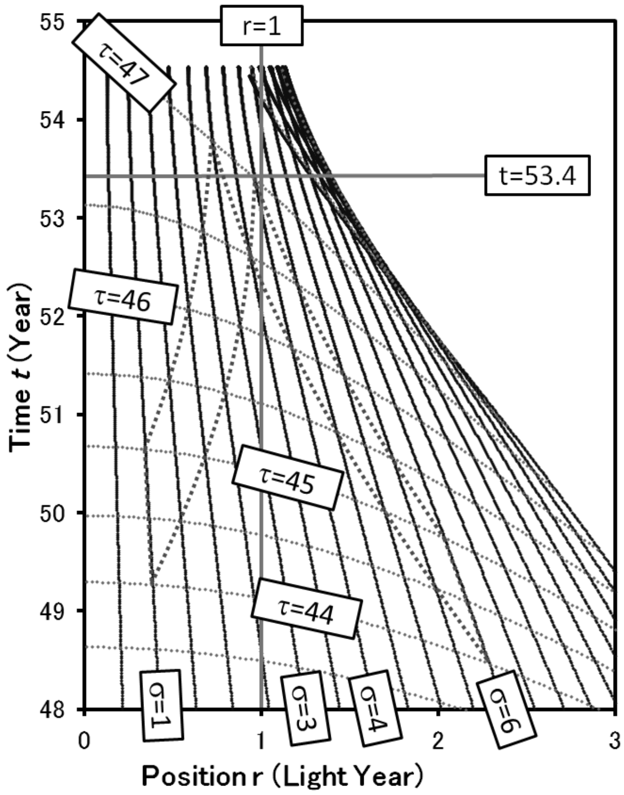

FIG. 3. Close-up of Fig. 2.

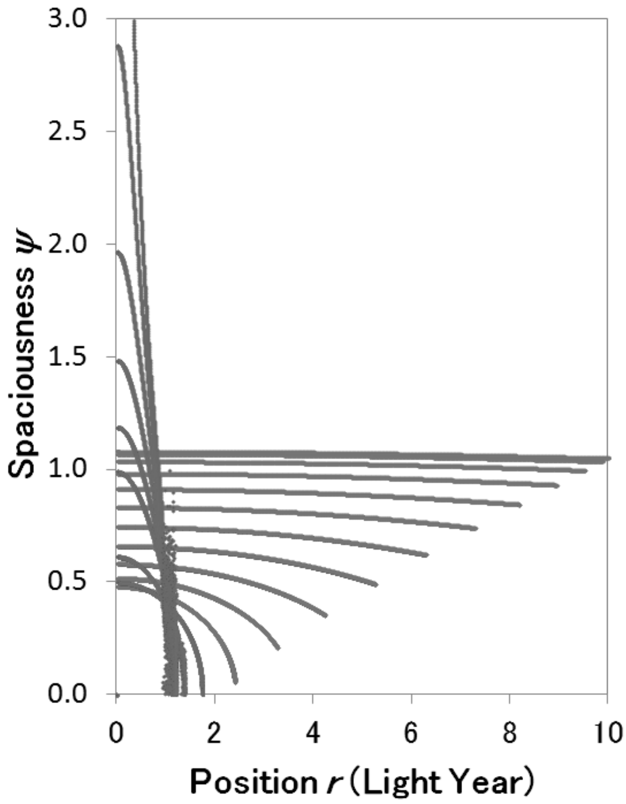

FIG. 4. Transition of spaciousness Ψ of the same simulation. Time interval between each curve is not constant.

Both light arrows reach between $\sigma = 3$ light year and $\sigma = 4$ light year at $\tau = 47$ year. There, length of both light arrows is exactly one local Lagrangian light year and N_G keeps the same. Therefore, in Lagrangian coordinates, wave length measured with the metric for light keeps the same value regardless of the direction the light came. Incidentally, by this time, size of the observer who is free-falling with dust shrinks because γ value at $\tau = 47$ year is larger than before. Consequently, for this observer, wave length of the light coming from both places are observed longer than original. This phenomena can explain cosmological red shift. If γ value is increasing linearly, wave length of the light from farer place is dilated more, proportional to the distance.

Strictly speaking, light came from inner place has shorter wave length than light from outer place with this hypothesis, because objects at inner place shrink earlier. However, this difference can be small enough if the original dust ball was large enough or the observer is not far from the center of the dust ball.

Besides, actually, cosmic microwave background radiation (CMB) is a little anisotropic. This anisotropy is interpreted as Doppler shift caused by our travelling speed against CMB. However, it is possible to interpret it as an indicator of the direction of the center of the universe.

Measured with the metric for matter, at place where γ is large and the scale shrinks, space between dust particles looks wider. Therefore, we defined spaciousness Ψ as follows.

$$\Psi = \gamma \frac{\Delta r}{\Delta r_i} \tag{18}$$

Where, Δr and Δr_i denotes current and initial radial distance between arbitrary two adjacent dust particles. In Fig. 4, transition of the distribution of spaciousness Ψ of the same simulation is shown. Each curve represents distribution of Ψ at each proper time τ. Time interval between each curve is not constant. At $t = \tau = 0$, Ψ curve spreads from $r = 0$ to $r = 10$ light year because the initial radius of the dust ball is 10 light years. After the release, radius of the sphere shrinks and Ψ decreases.

If this were a Newtonian simulation, intensity of gravitational acceleration is proportional to radius r. Consequently, it shrinks keeping the mass distribution similar to its original. And all of dust particles converge to $r = 0$ simultaneously to form a singular point. However, in this simulation, time proceeds faster at outer region. Therefore, outer region shrinks faster than inner region does. This makes outer region less spacious.

After a while, outermost dust particles catch up with next outermost dust particles. At this point, Ψ at the outermost dust particles becomes 0. And after this, dust particles of outer region begin to be accumulated around the outer edge and form a shell-like structure. This makes a similar situation as Fig. 1D. As the shell-like structure approaches to Schwarzschild radius, γ value in the sphere increases steeply and contracting velocity measured with Eulerian coordinates declines to very slow because of the slow time progress. Consequently, Ψ increases rapidly around the center of the shell.

In this simulation, Ψ increased to as large as thousands in wide region around the center before one variable in the program overflowed its double precision number limit. Final Ψ value differed with simulation condition like fineness of elements and time steps, whereas, blowing up always happened almost exactly at $t = 54.54$ year. The terminal time of Fig. 2, 3 is this time. Ψ increased rapidly near the terminal time of the simulation. From $t = 54.50$ year to 54.54 year, although radius of the dust ball shrank very little, γ and Ψ increased thousands times.

This simulation is intended for the evolution of our universe. However, it can also be considered as that of a black hole. It is often said that in black hole, matter falls into a singular point where the density is infinite. However, according to this simulation, we could find nothing special around the center. To the contrary, the center region of the dust ball became more and more spacious at an accelerating pace. This can explain so-called dark energy.

This simulation result also suggests that a black hole is not likely to be formed in finite time span. When a dust ball approaches to be a black hole, time progress inside the dust ball measured with the metric for light approaches to halt resisting to be a black hole. All so-called black holes may well be called *quasi-black holes.*

Furthermore, with this hypothesis, we have once experienced congested age before becoming spacious. This age makes the evolution similar to that of big bang theory after the inflation. CMB and nucleosynthesis can also be explained same as the standard theory.

Although Extreme γ value is a hurdle for simulation, it is not seem to be a hurdle to stop the time progress of real world. It is not sufficient to decide whether Eulerian time continues forever without having the big crunch. However, at least, Lagrangian time measured with the metric for matter seems to last forever because matter inside it can become infinitely small. We will have the big rip with this hypothesis.

Discussion about simulation code

Strictly speaking, results after overtaking are not correct by two reasons. First, this code is based on the premise that everything stays still with Lagrangian coordinates. Overtaking violates this premise. Second, this code is based on the premise that Birkhoff's theorem is valid. Whereas, that theorem is valid only if gravity works without time lag. In this case, since universe is very large, dust ball is not spherical observed from an observer free-falling with matter around the edge of it, because there is a large time lag between the observer's current time and the time gravitational pull was exerted by the matter at the other side of the dust ball.

For overcoming this second problem, we must employ simulation code which take account of propagation speed of gravity. However, it must be very difficult because propagation speed of gravity differs from place to place and gravitational force line must refract in the matter field.

Although our simple code is not strictly correct as stated above, as long as overtaking is not too large, we believe it is sufficient for us to verify our hypothesis.

CONCLUSIONS

- We have proposed a new interpretation of GR. With it;
  - Bona-fide Einstein field equations are used without any modification at all.
  - A new metric *metric for matter* is introduced. Any object including observer shrink at place where γ value is large. The metric for matter is the metric for shrunken observer. With it, space is observed wider.
  - Principle of equivalence and principle of energy conservation are both naturally satisfied without employing energy-momentum pseudotensor.
- We have proposed a new hypothesis *Free-falling dust ball universe.* It states;
  - The universe was born as a small particle by quantum mechanism. Physical rules were also created at this point. Measuring the universe with these new rules, the new-born universe was turned out to be a huge homogeneous dust ball.
  - Dust which represent matter began free-falling toward its center of gravity at the time of creation. At first, it shrank keeping the mass distribution similar to its original. After a while, because time proceeded faster at outer region, mass were concentrated at outer edge of the sphere to form a high density shell-like structure.
  - As the shell converged further to approach Schwarzschild radius, γ value inside the shell increased at an accelerating pace. It made spaciousness Ψ larger rapidly. This can explain observed accelerating expansion of the universe.

– G constant must be increasing in our world.

– Although there are celestial bodies so-called black holes, it is not likely that a black hole in the strict sense has ever been formed. Moreover, there are no singular points nor anything special around the centers of those celestial bodies. On the contrary, for observers inside them, density of matter becomes thinner and thinner as they approach to be black holes.

- The moon is gaining angular momentum more than the lost angular momentum of the earth. The momentum difference corresponds to the moon's rise of $0.76 cm/year$ and expansion rate of $2.0 \times 10^{-11} year^{-1}$.

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